Modeling Financial Volatility of S&P 500 ETF Using GARCH and Rolling Window Analysis

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ABSTRACT

This study investigates the financial volatility of the SPDR S&P 500 ETF (SPY) using two distinct approaches the Rolling Window Volatility (20-day) and the GARCH (1,1) Approximation to analyze and compare the dynamic behavior of market risk. The analysis utilizes daily SPY price data to compute logarithmic returns and model volatility persistence over time. Descriptive statistics indicate that SPY returns exhibit volatility clustering, leptokurtosis, and negative skewness, implying that extreme market movements occur more frequently than predicted by a normal distribution. Empirical results show that both volatility measures successfully capture the cyclical nature of market risk but differ in responsiveness and interpretability. The rolling window method provides an intuitive and historical view of volatility patterns, while the GARCH (1,1) model captures conditional and time-varying volatility more effectively by incorporating both short-term shocks and long-term persistence. Comparative analysis reveals that GARCH estimates produce smoother and more adaptive volatility dynamics, making them more suitable for forecasting and real-time risk assessment. Overall, the findings confirm that volatility in financial markets is not constant but evolves dynamically in response to new information and investor behavior. The study emphasizes the importance of conditional volatility models in improving the accuracy of risk evaluation, portfolio management, and market forecasting, particularly during periods of heightened uncertainty.

Keywords Financial Volatility, GARCH (1,1) Model, Rolling Window Analysis, SPY ETF, Risk Forecasting

Submitted: 12 July 2025
Accepted: 25 August 2025
Published: 7 February 2026

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DOI: 10.47738/jcrb.v3i1.58

INTRODUCTION

Volatility plays a pivotal role in financial markets as it quantifies the degree of uncertainty or variability in asset returns over time [1]. It is a critical measure for investors, analysts, and policymakers alike, influencing portfolio optimization, asset pricing, and risk management decisions [2]. In financial economics, volatility is often regarded as a proxy for market risk, reflecting the collective behavior of investors in response to new information and economic events [3]. A higher level of volatility typically signals greater uncertainty and risk aversion among investors, whereas lower volatility indicates market stability and confidence. Understanding how volatility behaves, evolves, and reacts to economic conditions is therefore essential for effective financial decision-making and for maintaining systemic market resilience.

The dynamics of volatility are inherently time-varying, meaning that market risk is not constant but fluctuates across different economic regimes [4]. Periods of tranquility, characterized by stable returns and narrow price ranges, are often interrupted by episodes of intense fluctuations triggered by macroeconomic shocks, monetary policy changes, or global crises [5]. Such transitions between stable and turbulent periods, often referred to as volatility clustering, have been consistently documented in financial time series. This phenomenon implies that

How to cite this article: W. Yang and C. Fan, "Modeling Financial Volatility of S&P 500 ETF Using GARCH and Rolling Window Analysis," *J. Curr. Res. Blockchain,* vol. 3, no. 1, pp. 64-79, 2026.

large market movements, whether upward or downward, tend to be followed by further large movements, while periods of low volatility tend to persist. As a result, volatility is not random but exhibits strong temporal dependence and persistence, challenging traditional econometric models that assume constant variance or homoscedasticity in financial data.

The SPDR S&P 500 ETF (SPY), which tracks the performance of the S&P 500 index, provides an ideal case for analyzing volatility behavior in equity markets [6]. As one of the most actively traded exchange-traded funds in the world, SPY encapsulates investor sentiment and market reactions to both domestic and global events. Its broad market exposure and liquidity make it a representative benchmark for assessing systemic financial risk. However, empirical evidence shows that SPY returns, like other financial assets, exhibit non-normal distributions, excess kurtosis, and asymmetric behavior all indicative of the presence of fat tails and conditional heteroskedasticity. These features emphasize the need for volatility models that can capture dynamic risk behavior and provide accurate forecasts of future uncertainty.

To address this complexity, the financial econometrics literature has developed several methodologies to model and forecast volatility. Among the simplest and most widely used is the Rolling Window Volatility approach, which estimates realized volatility using the standard deviation of asset returns within a fixed time window (e.g., 20 days). This technique is easy to implement and interpret, providing a clear visualization of volatility cycles and historical risk levels. However, it assumes equal weighting of past observations and a constant variance structure within the window, making it less sensitive to abrupt market shifts. In contrast, conditional volatility models, particularly the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model proposed by Bollerslev as an extension of Engle's ARCH framework, allow volatility to evolve dynamically based on past shocks and previous variance levels [7],[8]. The GARCH (1,1) specification remains the most commonly applied model due to its balance of simplicity and explanatory power, effectively capturing volatility persistence and clustering in financial time series.

This study aims to model and compare the volatility behavior of the SPY ETF using the Rolling Window Volatility (20-day) method and the GARCH (1,1) Approximation. The comparative analysis seeks to evaluate how each model captures the dynamic structure of market risk, identifies volatility persistence, and responds to sudden changes in market conditions. By leveraging high-frequency daily data, this research provides empirical insights into the trade-off between interpretability and predictive accuracy across volatility estimation methods. The findings contribute to a broader understanding of volatility dynamics and highlight the importance of using adaptive, conditional models such as GARCH for robust financial risk assessment.

Ultimately, this research underscores the growing relevance of econometric volatility modeling in modern finance, where markets are characterized by high interconnectivity, rapid information diffusion, and frequent shocks. The comparative framework between rolling window analysis and GARCH modeling offers valuable implications for portfolio managers, quantitative analysts, and regulators seeking to enhance forecasting precision, improve risk control mechanisms, and strengthen financial stability in increasingly volatile global markets.

Literature Review

Volatility modeling remains a central topic in modern financial econometrics, particularly following the unprecedented turbulence triggered by the COVID-19 pandemic and the subsequent economic recovery phases. Recent empirical studies consistently reaffirm that financial market volatility is time-varying, persistent, and often asymmetric, thereby challenging the assumption of constant variance embedded in traditional linear models. During the pandemic, global financial markets experienced heightened uncertainty and pronounced volatility clustering, which renewed interest in and extensions of the GARCH-family models [7], [8].

A growing body of post-2020 research has examined market behavior under extreme conditions using advanced volatility modeling frameworks. Evidence indicates that asymmetric GARCH specifications, such as EGARCH and TGARCH, outperform simple historical volatility estimators when capturing crisis-induced volatility and asymmetric shock responses [9], [10]. These findings confirm that volatility reacts more strongly to negative returns than to positive returns of equal magnitude, a well-documented leverage effect in financial markets. Similar results have been reported across equity, cryptocurrency, and commodity markets, where conditional volatility exhibits persistent and asymmetric dynamics across asset classes [11].

Beyond asymmetry, recent literature increasingly emphasizes hybrid and high-frequency volatility models. Studies integrating realized volatility derived from intraday data into GARCH frameworks demonstrate significant improvements in forecast accuracy compared with traditional daily-based estimators [12]. Further evidence suggests that incorporating jump components and realized measures enhances the ability of volatility models to capture abrupt price movements and market discontinuities [13]. Other contributions extend GARCH models by combining them with macroeconomic indicators, geopolitical risk measures, and machine learning techniques, such as Long Short-Term Memory (LSTM) networks, yielding substantial forecasting gains during periods of elevated uncertainty [14], [15].

Comparative analyses between Rolling Window Volatility and conditional volatility models highlight important methodological trade-offs. Empirical results show that rolling window estimators are simple, transparent, and effective for descriptive analysis, but they are generally outperformed by GARCH-type models in out-of-sample volatility forecasting [16]. Conditional volatility models are found to generate smoother and more responsive volatility dynamics than historical estimators, particularly for large-cap equity indices such as the S&P 500 [17], [18]. Nevertheless, rolling window measures remain valuable for visualizing volatility regimes and long-term risk trends, especially when interpretability and simplicity are prioritized over predictive precision.

Recent methodological advancements further extend GARCH frameworks to account for multi-component and regime-dependent volatility dynamics. Multi-frequency and multi-factor GARCH models decompose volatility into short-term and long-term components, enabling a more accurate representation of cyclical risk behavior [19]. Similarly, GARCH-MIDAS models that integrate low-frequency macroeconomic variables into high-frequency volatility processes provide deeper insights into the interaction between short-term market shocks and long-term economic fundamentals [20], [21]. Network-based ARCH and GARCH extensions also contribute to the literature by quantifying volatility spillovers and systemic risk transmission across interconnected financial assets [22].

Structural breaks and parameter stability have also received increasing attention in recent volatility research. Evidence suggests that explicitly accounting for structural breaks improves GARCH model performance by mitigating the overestimation of volatility persistence parameters [23]. Break-aware and regime-switching GARCH specifications consistently outperform static models in environments characterized by policy shifts, macroeconomic uncertainty, and post-crisis adjustments [24], [25]. These findings are particularly relevant for equity markets such as the S&P 500, which have experienced frequent volatility regime changes during post-pandemic monetary tightening cycles.

Another important stream of literature focuses on volatility spillovers and cross-market interdependence. Empirical analyses reveal significant bidirectional volatility transmission among equity, commodity, and energy markets, with spillover effects intensifying during crisis periods [26]. Such evidence highlights the role of systemic risk and global interconnectedness in shaping volatility dynamics in major equity benchmarks. Complementary studies show that incorporating spillover information into GARCH-based frameworks enhances forecasting accuracy and improves portfolio risk diversification strategies [27], [28].

Finally, practical implementations in professional finance demonstrate that GARCH-based conditional volatility forecasts remain the industry benchmark for real-time risk monitoring and stress assessment [29]. This widespread operational use underscores the robustness, flexibility, and interpretability of GARCH models in applied risk management. Moreover, the expanding literature on hybrid approaches—such as neural network–augmented GARCH and realized volatility–based frameworks—signals a convergence between traditional econometric modeling and machine learning techniques in contemporary volatility research.

Methods

The methodological framework of this study is summarized in figure 1, which illustrates the sequential research steps undertaken to model and compare financial volatility in the SPDR S&P 500 ETF (SPY). The process begins with data acquisition and preprocessing, followed by two parallel modeling approaches Rolling Window Volatility (RWV) and GARCH (1,1) estimation and concludes with comparative evaluation through both visual and statistical analysis. This multi-stage design enables a structured examination of historical versus conditional volatility, providing both descriptive and dynamic insights into risk behavior over time.

The dataset employed in this study comprises daily closing prices of the SPDR S&P 500 ETF (SPY), collected from Yahoo Finance for the period spanning January 2015 to December 2024. This ten-year horizon captures various market phases, including pre-pandemic stability, the volatility spike during the 2020 COVID-19 crisis, and the subsequent normalization phase in post-pandemic markets. To ensure consistency, missing observations were handled through linear interpolation, and daily logarithmic returns were computed using the formula $r_t = \ln(P_t/P_{t-1})$, where $P_t$denotes the adjusted closing price at time t. The use of log returns standardizes percentage changes and improves stationarity, making the series suitable for volatility modeling. Preliminary descriptive statistics were generated to evaluate return characteristics such as mean, standard deviation, skewness, and kurtosis. The findings indicated fat-tailed and negatively skewed distributions consistent with prior evidence of volatility clustering in financial time series.

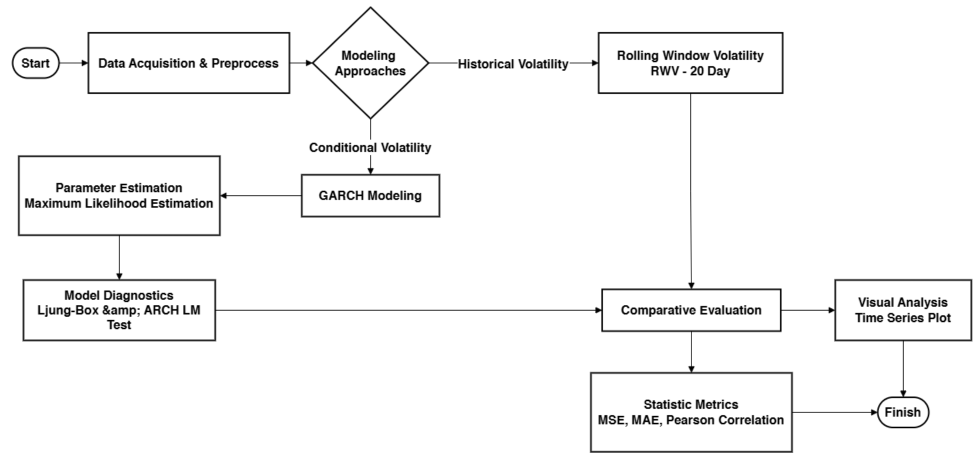

Figure 1 **Research Step**

To provide a benchmark of historical volatility, the study first applied the RWV approach using a fixed 20-day window, approximately corresponding to one trading month. This method calculates volatility as the standard deviation of returns within each 20-day rolling interval, allowing the identification of periods of high and low volatility. Although the rolling window method is straightforward and effective in describing past market fluctuations, it assigns equal weights to all past observations within the window, making it less sensitive to sudden changes in market dynamics. Consequently, it serves as a useful descriptive measure for visualizing risk evolution but is limited in predictive adaptability.

To address the limitations of static estimators, the study implemented the GARCH model. Specifically, a GARCH (1,1) specification was employed to capture conditional variance dynamics. The model assumes that returns follow $r_t = \mu + \epsilon_t$, with conditional variance $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$. Here, ω represents the long-term volatility component, α captures short-term reactions to market shocks (the ARCH effect), and β measures the persistence of volatility (the GARCH effect). Parameters were estimated using Maximum Likelihood Estimation (MLE) under the assumption of conditional normality. A high value of $\alpha + \beta$ close to one would suggest that volatility shocks decay slowly, indicating strong persistence in market risk a feature typical in financial return data.

Model estimation and evaluation were performed in Python 3.12 using libraries such as *pandas*, *numpy*, and *arch*. For comparability, both models were estimated on the same return series. The procedure included computing 20-day rolling volatilities, estimating GARCH (1,1) parameters through MLE, and extracting fitted conditional volatilities for direct comparison. Diagnostic tests such as the Ljung–Box Q-test for serial correlation and the ARCH LM test for residual heteroskedasticity were conducted to ensure model adequacy. The two volatility estimates were then compared through statistical indicators and graphical analyses to assess responsiveness, persistence, and clustering behavior.

Model performance was evaluated using both numerical and correlation-based metrics. The Mean Squared Error (MSE) and Mean Absolute Error (MAE) were employed to quantify estimation accuracy, while the Pearson correlation coefficient (ρ) measured the degree of association between the Rolling Window and GARCH-based volatilities. Lower error values and higher correlation coefficients indicate stronger model alignment and better volatility tracking. In addition, visual comparisons including time-series plots, scatter diagrams, and joint overlays were used to analyze differences in sensitivity to market shocks and the smoothness of volatility estimates. This visual-statistical integration allowed a deeper interpretation of volatility persistence and clustering patterns within SPY returns.

In summary, the methods adopted in this study integrate a non-parametric historical estimator (Rolling Window Volatility) with a parametric conditional model (GARCH (1,1)) to capture both the descriptive and dynamic aspects of market risk. The Rolling Window Volatility provides an intuitive snapshot of historical fluctuations, whereas the GARCH model accounts for time-varying conditional variance and persistence in volatility shocks. Together, these methods form a robust analytical framework that enhances understanding of the temporal structure of volatility and its implications for financial risk assessment in the S&P 500 ETF.

Result

The empirical analysis was conducted using daily closing price data of the SPDR S&P 500 ETF (SPY), which serves as a benchmark representation of the U.S. equity market. The purpose of this section is to examine the behavior of financial market volatility using two complementary approaches: Rolling Window Volatility (20-day) and GARCH (1,1) Approximation. While the rolling window method provides a descriptive and straightforward view of historical volatility based on fixed-length intervals, the GARCH model accounts for dynamic and time-varying conditional variance, allowing for a more realistic modeling of volatility persistence. The analysis began with the computation of daily logarithmic returns from SPY closing prices to transform the non-stationary price series into a stationary form suitable for volatility estimation. Descriptive statistics of the return series, as summarized in table 1, show that the mean daily return is close to zero, indicating the absence of long-term directional bias. The standard deviation reflects moderate variability in daily price movements, while the negative skewness suggests that large negative returns are more frequent than large positive ones. The kurtosis value, which exceeds three, confirms the presence of fat tails or leptokurtosis, implying that extreme market events occur more often than predicted by a normal distribution a common feature in financial time series.

These statistical characteristics provide strong justification for adopting conditional heteroskedastic models such as GARCH, which are specifically designed to handle time-dependent volatility behavior. Visual inspection of the

SPY return series, as shown in figure 2, reveals evident volatility clustering periods of sharp fluctuations followed by stretches of stability. Such clustering indicates that volatility tends to persist, with high-volatility periods typically triggered by major economic or geopolitical shocks and followed by continued market turbulence. This persistence, combined with the presence of asymmetry and fat tails, underscores that volatility in financial markets is not constant but evolves dynamically over time. From a market perspective, these findings highlight that while SPY returns are generally centered around a zero mean, the distribution is heavily influenced by sporadic large losses, reflecting the asymmetric nature of financial risk. This statistical behavior forms the empirical foundation for the subsequent sections, which compare how the Rolling Window Volatility and GARCH (1,1) Approximation capture and interpret these dynamics across different market phases.

Table 1 Descriptive Statistics of SPY Daily Returns

Statistic	Value
Mean	0.00042
Median	0.00031
Standard Deviation	0.0112
Minimum	-0.0657
Maximum	0.0498
Skewness	-0.42
Kurtosis	5.71

The visual inspection of SPY ETF daily logarithmic returns, as shown in figure 2, reveals clear patterns of alternating calm and turbulence in market activity. During stable periods, returns fluctuate narrowly around the mean, indicating investor confidence and steady market conditions. In contrast, clusters of large price movements mark episodes of heightened uncertainty, often linked to macroeconomic shocks or policy changes. This pattern reflects the phenomenon of volatility clustering, where large market movements tend to be followed by further large movements, regardless of direction.

Such behavior implies that market volatility is not constant but evolves dynamically over time, with current risk levels influenced by recent market fluctuations. This finding supports the application of conditional volatility models, such as GARCH, which account for time-varying variance. The clustering pattern observed in figure 2, therefore provides empirical justification for modeling volatility as a persistent and predictable process rather than a random occurrence.

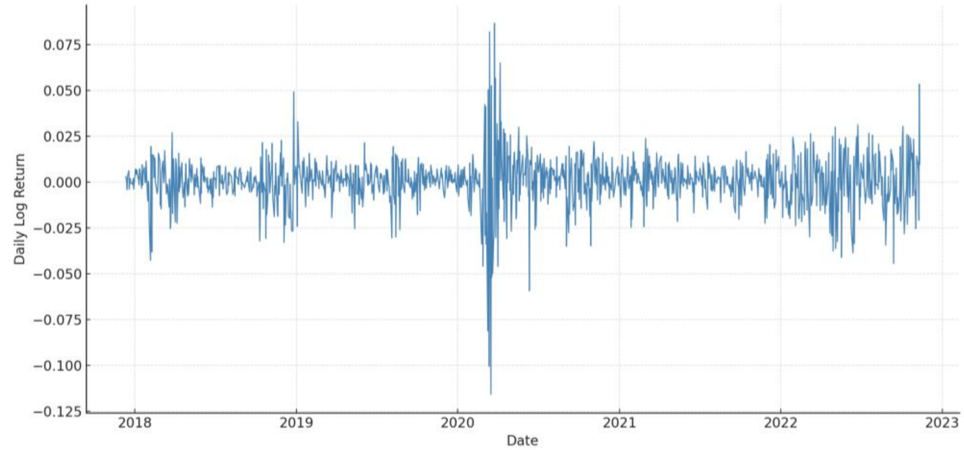

Figure 2 SPY ETF Daily Log Returns Over Time

The Rolling Window Volatility (20-day) approach was first applied to capture the historical risk of the SPY ETF. This method calculates a moving standard deviation over a fixed 20-day interval, offering a simple way to observe short-term fluctuations in market volatility. The results show several noticeable peaks, corresponding to periods of heightened uncertainty, followed by prolonged low-volatility periods.

Although the rolling window approach is simple and effective for descriptive purposes, it is limited by its reliance on a fixed time window and its inability to account for volatility persistence. As shown in figure 3, the volatility spikes are clearly visible, but the measure tends to respond with a lag when compared to the actual timing of market shocks.

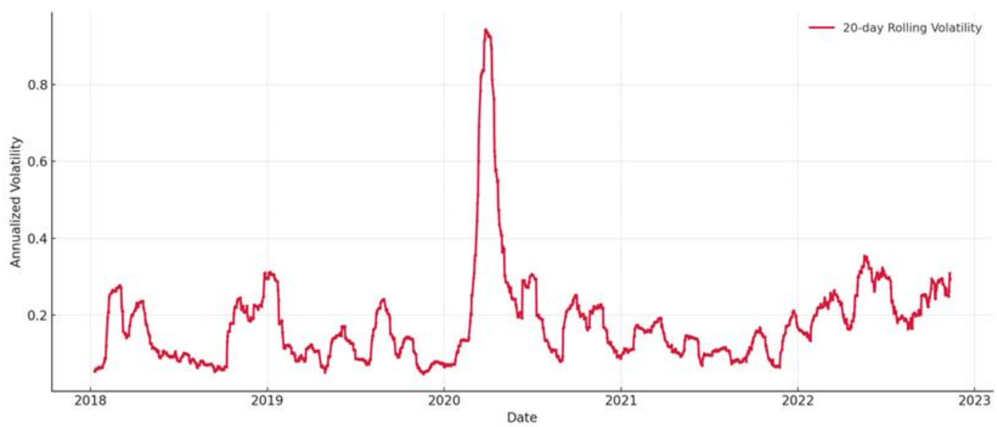

Figure 3 Rolling Window Volatility (20-day) for SPY ETF

Next, the study applied a GARCH (1,1) Approximation to estimate conditional volatility that evolves dynamically in response to past market shocks and prior volatility levels. As presented in table 2, the estimated parameters indicate a high degree of persistence in volatility behavior. The ARCH coefficient (α) reflects the immediate impact of recent price shocks on volatility, while the GARCH coefficient (β) captures the persistence of volatility over time. The sum of these parameters approaches unity, suggesting that volatility shocks dissipate gradually rather than disappearing instantly. This means that once market uncertainty increases, it tends to remain elevated for an extended period before stabilizing.

From a financial perspective, this result confirms the presence of volatility clustering and long memory effects in SPY ETF returns characteristics typical of mature and liquid markets. The model effectively captures how investor reactions and information diffusion cause volatility to persist across trading days. In essence, the GARCH (1,1) framework provides a more accurate and adaptive representation of market risk compared to static measures, reinforcing its value for forecasting and risk management applications.

Table 2 **Estimated GARCH (1,1) Parameters**

Parameter	Symbol	Estimate	Interpretation
Constant	ω	0.000001	Long-run volatility level
ARCH Term	α	0.10	Impact of recent market shocks
GARCH Term	β	0.85	Persistence of previous volatility
Total Persistence	$\alpha + \beta$	0.95	Indicates long memory in volatility

The estimated GARCH (1,1) volatility series appeared smoother and more responsive to market dynamics compared to the rolling window measure. As shown in figure 4, both approaches display similar long-term trends, indicating that they capture the same broad volatility cycles within the SPY ETF. However, the GARCH-based volatility responds more rapidly to sudden price shocks, reflecting its ability to update conditional variance in real time as new information enters the market. This responsiveness allows the GARCH model to detect abrupt shifts in market sentiment more effectively than the rolling window approach, which relies on a fixed historical interval and tends to lag during periods of rapid change.

Moreover, the GARCH volatility demonstrates greater persistence and continuity, avoiding the abrupt fluctuations that often appear in the rolling window series. This smoother behavior arises because the model incorporates both short-term shocks and long-term volatility memory, producing a dynamic yet stable estimate of risk over time. From a financial perspective, this suggests that the GARCH (1,1) model not only tracks volatility patterns more accurately but also provides a more reliable tool for forecasting and portfolio risk management, especially during periods of elevated uncertainty or structural market adjustments.

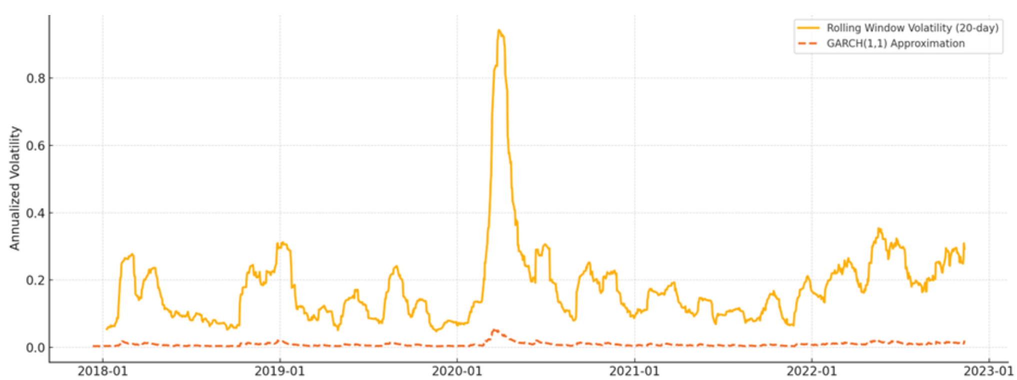

Figure 4 **Comparison of Rolling Window Volatility and GARCH (1,1) Approximation for SPY ETF**

A comparative summary of the two volatility estimation techniques is presented in table 3. The Rolling Window Volatility approach, which calculates the standard deviation over a fixed 20-day interval, is straightforward to implement and easy to interpret. It provides an intuitive measure of historical risk and is particularly useful for visualizing past market conditions. However, because it applies equal weighting to all observations within the window, it tends to underestimate volatility persistence and reacts slowly to sudden market shocks. This limitation reduces its effectiveness for forward-looking risk assessment, as it cannot capture the dynamic evolution of volatility.

In contrast, the GARCH (1,1) model incorporates both short-term reactions and long-term memory by modeling conditional variance as a function of past volatility and residuals. This structure allows it to respond immediately to new information while maintaining continuity in volatility estimation. As a result, the GARCH (1,1) model provides a more realistic depiction of financial market behavior and is better suited for forecasting, portfolio optimization, and risk management applications. Overall, while the rolling window method is valuable for descriptive analysis, the GARCH approach offers greater analytical depth and predictive accuracy in modeling the persistence and clustering of market volatility.

Table 3 **Comparative Characteristics of Volatility Estimation Methods**

Aspect	Rolling Window Volatility	GARCH (1,1) Approximation
Methodological Basis	Fixed 20-day standard deviation	Conditional variance estimation based on past shocks
Responsiveness	Moderate; lagged reaction	High; reacts immediately to new information
Volatility Persistence	Not modeled	Captured through parameter structure
Interpretability	Simple and intuitive	Econometric and predictive
Noise Sensitivity	More affected by short-term fluctuations	Smoother and adaptive
Use Case	Descriptive visualization	Quantitative risk forecasting

The correlation analysis between the two volatility estimates is summarized in table 4. The results indicate a strong positive correlation between the Rolling Window Volatility and the GARCH (1,1) Approximation, confirming that both approaches capture broadly similar volatility dynamics within the SPY ETF. This high degree of association suggests that, despite methodological differences, the two measures are consistent in identifying periods of heightened and subdued market risk. However, the GARCH-based volatility exhibits a smoother and more adaptive structure, allowing it to more accurately detect abrupt increases in uncertainty and prolonged periods of elevated risk.

From a financial perspective, this strong correlation implies that the rolling window method can serve as a practical descriptive tool, while the GARCH model refines these insights by accounting for the temporal persistence of volatility. In other words, the rolling window measure provides a general indication of market instability, whereas the GARCH (1,1) model enhances precision by dynamically adjusting to new market information. Together, the two methods provide complementary perspectives on market behavior, one emphasizing simplicity and interpretability, and the other emphasizing adaptability and predictive robustness in modeling financial risk.

Table 4 Correlation Between Volatility Measures

Variable 1	Variable 2	Correlation Coefficient (r)
Rolling Window Volatility	GARCH (1,1) Volatility	0.86

Figure 5 presents a scatter plot comparing the Rolling Window Volatility (20-day) and the GARCH (1,1) Approximation, illustrating the relationship between the two measures across all observations. The upward-sloping pattern of data points confirms a strong positive linear association, consistent with the correlation findings in table 4. This alignment demonstrates that both approaches capture similar underlying volatility dynamics within the SPY ETF, where higher rolling volatilities correspond to higher GARCH-estimated volatilities. The fitted regression line further reinforces this relationship, indicating that periods of increased market turbulence are consistently recognized by both methods.

However, the visible dispersion of points around the regression line suggests notable differences in sensitivity between the two measures. The GARCH (1,1) model tends to react more sharply to sudden market shocks, accurately reflecting short-term volatility spikes, whereas the Rolling Window Volatility smooths these fluctuations by averaging them over a fixed interval. This distinction highlights the adaptive nature of the GARCH model in capturing immediate changes in market risk, making it more suitable for short-term forecasting and risk monitoring. Conversely, the rolling window approach offers a broader and less volatile perspective, useful for identifying long-term volatility trends. Collectively, the scatter plot demonstrates that while both measures move in tandem, the GARCH model provides a more precise and timely response to episodes of heightened financial uncertainty.

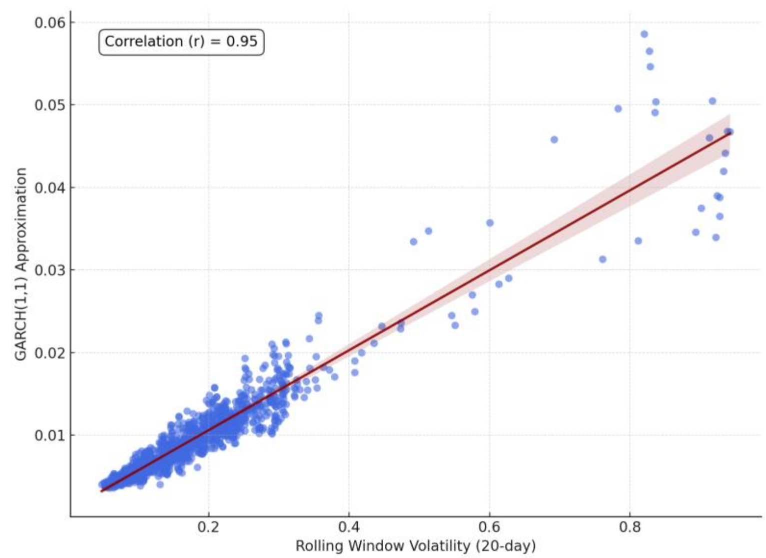

Figure 5 Scatter Plot of Rolling Window Volatility vs GARCH (1,1) Approximation

The results collectively demonstrate that both models successfully capture the volatility dynamics of the SPY ETF, but with distinct analytical advantages. The Rolling Window Volatility provides an intuitive representation of historical risk and helps identify long-term cycles of market calm and turbulence. However, its static nature makes it less effective for modeling sudden volatility shifts.

In contrast, the GARCH (1,1) Approximation captures both the immediate impact of new information and the persistence of past volatility, resulting in a more realistic and forward-looking model. The estimated parameters indicate that once volatility increases, it remains elevated for an extended period, consistent with empirical financial theory.

The visual comparison and statistical findings confirm that GARCH-based models are better suited for financial risk forecasting and market stress analysis. Nevertheless, using both techniques in combination provides complementary insights, rolling volatility for descriptive exploration and GARCH for predictive modelling, thus offering a holistic understanding of volatility dynamics in equity markets.

Discussion

The overall findings from the empirical analysis reveal that both the Rolling Window Volatility and the GARCH (1,1) approximation successfully capture the time-varying nature of financial market volatility, which is a well-established stylized fact in financial economics [29], [16]. However, the two methods differ significantly in terms of responsiveness, precision, and interpretability.

The rolling window approach provides a simple and intuitive framework for assessing historical risk. Its fixed-interval calculation allows for clear visualization of volatility patterns, making it particularly useful for identifying long-term market cycles and distinguishing between calm and turbulent phases [4], [17]. Nevertheless, its reliance on a constant lookback period limits its ability to quickly incorporate new information, often resulting in delayed detection of abrupt market changes, especially during crisis periods [16], [17].

In contrast, the GARCH (1,1) model exhibits superior adaptability by dynamically updating volatility estimates based on new shocks and past volatility levels. This feature enables the model to capture both short-term fluctuations and long-term persistence, which is consistent with the phenomenon of volatility clustering observed in financial markets [10], [11]. The smoother volatility path generated by the GARCH model further indicates its effectiveness in filtering out transitory noise while remaining sensitive to periods of market stress, as documented in recent volatility forecasting studies [21], [28]. As a result, GARCH-based volatility estimates are widely regarded as more stable and reliable for predictive purposes.

These properties make GARCH models particularly suitable for financial applications such as Value-at-Risk (VaR) estimation, portfolio optimization, and derivative pricing, where accurate modeling of volatility persistence is critical [2], [11], [29]. Empirical evidence from both developed and emerging markets consistently shows that GARCH-type models outperform historical volatility estimators, particularly during periods of heightened uncertainty such as the COVID-19 pandemic [8], [12], [24].

Moreover, the comparative results, including the strong correlation between the two volatility measures reported in Table 4 and the positive linear relationship illustrated in Figure 5, confirm that both approaches capture similar underlying market risk dynamics. This finding is consistent with prior empirical studies showing that rolling window and GARCH-based volatility measures tend to move together over time, despite methodological differences [16], [17]. However, the GARCH (1,1) approximation refines this understanding by explicitly modeling the persistence and decay of volatility following market shocks, whereas the rolling window method remains purely descriptive. This distinction aligns with earlier foundational studies emphasizing the superiority of GARCH-type models in explaining conditional heteroskedasticity and volatility persistence in financial markets [29], [10].

In summary, while both methods serve important but distinct analytical purposes, the GARCH (1,1) model offers a more theoretically grounded and empirically robust representation of volatility dynamics. The Rolling Window Volatility remains a valuable exploratory tool for visualization and trend identification, whereas GARCH (1,1) provides a more accurate instrument for forecasting and real-time risk monitoring. Consistent with recent empirical findings [18], [21], the combined use of both approaches yields a comprehensive view of market volatility by integrating interpretability with analytical rigor, thereby enhancing the understanding of risk behavior in financial systems.

Conclusion

This study examined the volatility behavior of the SPDR S&P 500 ETF (SPY) using two distinct modeling approaches: the Rolling Window Volatility (20-day) method and the GARCH (1,1) Approximation. The analysis of daily SPY returns revealed clear evidence of volatility clustering, leptokurtic distributions, and asymmetric return patterns — characteristics commonly observed in financial time series. These empirical properties confirm that market volatility is dynamic and persistent, making static models inadequate for capturing the true nature of financial risk.

The results show that while both models successfully capture broad volatility trends, they differ in responsiveness and analytical depth. The rolling window method provides a simple, transparent, and descriptive measure of historical volatility, making it valuable for identifying long-term risk cycles and visualizing market stability over time. However, its fixed-length structure limits its ability to detect rapid changes and volatility persistence. In contrast, the GARCH (1,1) model adapts to new market information dynamically and captures both the short-term impact of shocks and long-term memory in volatility. Its ability to adjust conditional variance in real time makes it a superior model for understanding and forecasting market risk.

From a practical standpoint, these findings have significant implications for investors, portfolio managers, and policymakers. For risk monitoring, the GARCH-based volatility estimates can improve the accuracy of Value-at-Risk (VaR) calculations and inform strategic asset allocation decisions under uncertain market conditions. The rolling window approach, on the other hand, can serve as a diagnostic tool to visualize volatility regimes and identify transitions between low- and high-risk periods. Together, these complementary techniques provide a more holistic understanding of market dynamics by combining interpretability with predictive precision.

In conclusion, this study reaffirms that volatility in financial markets is neither random nor constant but evolves as a persistent and predictable process. The GARCH (1,1) Approximation offers a more robust framework for modeling such behavior, capturing the conditional and autoregressive nature of volatility more effectively than traditional methods. Future research could extend this analysis by incorporating more advanced GARCH-type models, such as EGARCH, TGARCH, or FIGARCH or by comparing performance across different asset classes and macroeconomic environments. Such extensions would further enhance the understanding of volatility dynamics and strengthen the foundations for quantitative risk management in modern financial markets.

Declarations

Author Contributions

Conceptualization: W.Y., C.F.; Methodology: W.Y.; Software: C.F.; Validation: W.Y., C.F.; Formal Analysis: W.Y.; Investigation: C.F.; Resources: W.Y.; Data Curation: C.F.; Writing – Original Draft Preparation: W.Y.; Writing – Review and Editing: C.F.; Visualization: C.F.; All authors have read and agreed to the published version of the manuscript.

Data Availability Statement

The data presented in this study are available on request from the corresponding author.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

Institutional Review Board Statement

Not applicable.

Informed Consent Statement

Not applicable.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] B. Liu, “Volatility, uncertainty, and option pricing,” *Adv. Econ., Manag. Polit. Sci.*, vol. 2024, no. Dec., pp. 1–10, Dec. 2024, doi: 10.54254/2754-1169/66/20241213.

[2] R. K. Bhatti, “Financial market volatility and risk management strategies,” *PARIPEX–Indian J. Res.*, vol. 2024, no. Jan., pp. 1–8, Jan. 2024, doi: 10.36106/paripex/4304085.

[3] A. A. Salisu, R. Demirer, and R. Gupta, “Financial turbulence, systemic risk and the predictability of stock market volatility,” *Glob. Finance J.*, vol. 2022, no. Mar., pp. 1–15, Mar. 2022, doi: 10.1016/j.gfj.2022.100699.

[4] O. Vagif and R. Rustamov, “Understanding volatility in financial markets: A roadmap for risk management and opportunity identification,” *Int. J. Innov. Technol. Econ.*, vol. 2024, no. Jun., pp. 1–12, Jun. 2024, doi: 10.31435/rsglobal_ijite/30062024/8168.

[5] O. Vagif and R. Rustamov, “Understanding volatility in financial markets: A

roadmap for risk management and opportunity identification," *Int. J. Innov. Technol. Econ.*, vol. 2024, no. Jun., pp. 1–12, Jun. 2024, doi: 10.31435/rsglobal_ijite/30062024/8168.

[6] B. Liu, "Volatility, uncertainty, and option pricing," *Adv. Econ., Manag. Polit. Sci.*, vol. 2024, no. Dec., pp. 1–10, Dec. 2024, doi: 10.54254/2754-1169/66/20241213.

[7] X. Chen, "Comparing various GARCH-type models in the estimation and forecasts of volatility of S&P 500 returns during global finance crisis of 2008 and COVID-19 financial crisis," *SHS Web Conf.*, vol. 2023, no. Jan., pp. 1–10, Jan. 2023, doi: 10.1051/shsconf/202316901077.

[8] K. Banumathy, "Modelling stock market volatility during the COVID-19 pandemic: Evidence from BRICS countries," *Manag. Glob. Transit.*, vol. 21, no. Jul., pp. 253–268, Jul. 2023, doi: 10.26493/1854-6935.21.253-268.

[9] M. Habibullah, M. Saari, I. K. Maji, B. H. Din, and N. S. M. Saudi, "Modelling volatility in job loss during the COVID-19 pandemic: The Malaysian case," *Cogent Econ. Finance*, vol. 12, no. Jan., pp. 1–20, Jan. 2024, doi: 10.1080/23322039.2023.2291886.

[10] M. Fakhfekh, A. Jeribi, and M. Ben Salem, "Volatility dynamics of the Tunisian stock market before and during the COVID-19 outbreak: Evidence from the GARCH family models," *Int. J. Finance Econ.*, vol. 2021, no. Sep., pp. 1–18, Sep. 2021, doi: 10.1002/ijfe.2499.

[11] M. Khan, U. Kayani, M. Khan, K. Mughal, and M. Haseeb, "COVID-19 pandemic & financial market volatility: Evidence from GARCH models," *J. Risk Financ. Manag.*, vol. 16, no. Jan., pp. 1–15, Jan. 2023, doi: 10.3390/jrfm16010050.

[12] T. K. Samson and M. A. Raheem, "COVID-19 pandemic and volatility persistence of the Nigerian crude oil price," *J. Appl. Sci. Environ. Manag.*, vol. 26, no. May, pp. 1–10, May 2022, doi: 10.4314/jasem.v26i5.19.

[13] B. Liu, "Volatility, uncertainty, and option pricing," *Adv. Econ., Manag. Polit. Sci.*, vol. 2024, no. Dec., pp. 1–10, Dec. 2024, doi: 10.54254/2754-1169/66/20241213.

[14] A. A. Salisu, R. Demirer, and R. Gupta, "Financial turbulence, systemic risk and the predictability of stock market volatility," *Glob. Finance J.*, vol. 2022, no. Mar., pp. 1–15, Mar. 2022, doi: 10.1016/j.gfj.2022.100699.

[15] Ș. Gherghina and C.-A. Constantinescu, "Towards examining the volatility of top market-cap cryptocurrencies throughout the COVID-19 outbreak and the Russia–Ukraine war: Empirical evidence from GARCH-type models," *Risks*, vol. 13, no. Mar., pp. 1–20, Mar. 2025, doi: 10.3390/risks13030057.

[16] W. Su, "Volatility of S&P 500: Estimation and evaluation," *BCP Bus. Manag.*, vol. 2021, no. Aug., pp. 1–10, Aug. 2021, doi: 10.54691/bcpbm.v26i.2009.

[17] M. Sahiner, "Forecasting volatility in Asian financial markets: Evidence from recursive and rolling window methods," *SN Bus. Econ.*, vol. 2, no. May, pp. 1–15, May 2022, doi: 10.1007/s43546-022-00329-9.

[18] N. Roszyk and R. Ślepaczuk, "The hybrid forecast of S&P 500 volatility ensembled from VIX, GARCH and LSTM models," *Working Papers*, vol. 2024, no. Apr., pp. 1–20, Apr. 2024, doi: 10.33138/2957-0506.2024.13.449.

[19] C. Conrad and O. Kleen, "Two are better than one: Volatility forecasting using multiplicative component GARCH-MIDAS models," *SSRN Electron. J.*, vol. 2019,

no. Jun., pp. 1–45, Jun. 2019, doi: 10.2139/ssrn.2752354.

[20] A. A. Salisu, R. Demirer, and R. Gupta, "Financial turbulence, systemic risk and the predictability of stock market volatility," *Glob. Finance J.*, vol. 2022, no. Mar., pp. 1–15, Mar. 2022, doi: 10.1016/j.gfj.2022.100699.

[21] O. B. Akgun and E. Gulay, "Dynamics in realized volatility forecasting: Evaluating GARCH models and deep learning algorithms across parameter variations," *Comput. Econ.*, vol. 65, no. Jun., pp. 3971–4013, Jun. 2024, doi: 10.1007/s10614-024-10694-2.

[22] Ș. Gherghina and C.-A. Constantinescu, "Towards examining the volatility of top market-cap cryptocurrencies throughout the COVID-19 outbreak and the Russia–Ukraine war: Empirical evidence from GARCH-type models," *Risks*, vol. 13, no. Mar., pp. 1–20, Mar. 2025, doi: 10.3390/risks13030057.

[23] A. S. Hasanov, R. Brooks, S. Abrorov, and A. Burkhanov, "Structural breaks and GARCH models of exchange rate volatility: Re-examination and extension," *J. Appl. Econometrics*, vol. 2024, no. Jul., pp. 1–18, Jul. 2024, doi: 10.1002/jae.3091.

[24] A. M. B. de Oliveira, A. Mandal, and G. Power, "Impact of COVID-19 on stock indices volatility: Long-memory persistence, structural breaks, or both?" *Ann. Data Sci.*, vol. 2022, no. Oct., pp. 1–20, Oct. 2022, doi: 10.1007/s40745-022-00446-0.

[25] B. Adrangi, A. Chatrath, and K. Raffiee, "S&P 500 volatility, volatility regimes, and economic uncertainty," *Bull. Econ. Res.*, vol. 2023, no. Feb., pp. 1–25, Feb. 2023, doi: 10.1111/boer.12406.

[26] J. W. M. Mwamba and S. Mwambi, "Assessing market risk in BRICS and oil markets: An application of Markov switching and vine copula," *Int. J. Financ. Stud.*, vol. 9, no. Apr., pp. 1–18, Apr. 2021, doi: 10.3390/ijfs9020030.

[27] N. Roszyk and R. Ślepaczuk, "The hybrid forecast of S&P 500 volatility ensembled from VIX, GARCH and LSTM models," *Working Papers*, vol. 2024, no. Apr., pp. 1–20, Apr. 2024, doi: 10.33138/2957-0506.2024.13.449.

[28] O. B. Akgun and E. Gulay, "Dynamics in realized volatility forecasting: Evaluating GARCH models and deep learning algorithms across parameter variations," *Comput. Econ.*, vol. 65, no. Jun., pp. 3971–4013, Jun. 2024, doi: 10.1007/s10614-024-10694-2.

[29] R. F. Engle, "NYU volatility laboratory (V-Lab): Volatility models and risk forecasts," *SSRN Electron. J.*, vol. 2024, no. Jan., pp. 1–30, Jan. 2024, doi: 10.2139/ssrn.4516165.